Jeitschko, W. (1975). Acta Cryst. B31, 1187-1190.
Johnson, V. (1975). Inorg. Chem. 14, 1117-1120.
Jost, K. H., Ziemer, B. \& Seydel, R. (1977). Acta Cryst. B33, 1696-1700.
Liu, L.-G. (1978). Phys. Chem. Miner. 3, 291-299.
Midgley, C. M. (1952). Acta Cryst. 5, 307-312.
Mikhail, I. \& Peters, K. (1979). Acta Cryst. B35, 1200-1201. Moore, P. B. (1973). Am. Mineral. 58, 32-42.
O'Keeffe, M. \& Hyde, B. G. (1977). Acta Cryst. B33, 38023813.

O'Keeffe, M. \& Hyde, B. G. (1984). Nature (London), 309, 411-414.
O'Keeffe, M. \& Hyde, B. G. (1985). Struct. Bonding (Berlin). In the press.

Regourd, M., Bigare, M., Forest, J. \& Guinier, A. (1969). Vth International Symposium on the Chemistry of Cement, Tokyo, 1968, Vol. 1, pp. 44-48. Tokyo: The Cement Association of Japan. Ringwood, A. E. \& Reid, A. F. (1968). Earth Planet. Sci. Lett. 5, 67-70.
SaAlfeld, H. (1975). Am. Mineral. 60, 824-827.
Saflfeld, R. \& Klaska, K. H. (1981). Z. Kristallogr. 154, 323-324; 155, 65-73.
Schubert, K. (1964). Kristallstrukturen zweikomponenter Phasen. Berlin: Springer-Verlag.
Smith, D. K., Majumdar, A. \& Ordway, F. (1965). Acta Cryst. 18, 787-795.
Udagawa, S., Urabe, K., Natsume, M. \& Yano, T. (1980). Cem. Concr. Res. 10, 139-144.

Acta Cryst. (1985). B41, 390-395

# 24-Layer Structure of Tricalcium Germanate, $\mathrm{Ca}_{3} \mathbf{G e O}_{5}$ 

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#### Abstract

The crystal structure of a 24-layer polytype of tricalcium germanate $\left(C_{3} G\right)$, which was found in a flux melt, has been determined. The crystal data are: $M_{r}=$ 272.8, rhombohedral, $R 3 m, a=7 \cdot 228(2), \quad c=$ 67.42(2) $\AA, \quad V=3050(2) \AA^{3}, \quad Z=24, \quad D_{x}=3.56$ $\mathrm{g} \mathrm{cm}^{-3}, \quad$ Мо $K \alpha, \quad \lambda=0.71069 \AA, \quad \mu=92.7 \mathrm{~cm}^{-1}$, $F(000)=3168, T=298 \mathrm{~K}$. The structure consists of two kinds of subunits, one is related to the structure of the 2-layer polytype and the other to that of the 9-layer polytype; these occur alternately in the structure. Half of the eight independent germanate tetrahedra show nearly perfect orientational disorder along the $c$ axis. The positions of the Ge atoms of these tetrahedra are split, the separations being in the range $0 \cdot 34(10)$ to $0 \cdot 69(5) \AA$. The coordination numbers of Ca , each calculated as the sum of the occupancies of the $\mathrm{Ca}-\mathrm{O}$ bonds, are in the range 6.0 to 6.56 , giving mean $\mathrm{Ca}-\mathrm{O}$ bond lengths of 2.34 to $2.51 \AA$. General principles of deriving polytypes for $\mathrm{C}_{3} \mathrm{G}$ are provided and, based on these, the stacking sequences of simple polytypes such as 4 -layer, 5-layer and 6-layer structures have been predicted. The derivation of polytypes can be applicable to the case of tricalcium silicate $\left(\mathrm{C}_{3} \mathrm{~S}\right)$.


## Introduction

Our study on tricalcium germanate $\left(\mathrm{C}_{3} \mathrm{G}\right)$ has revealed the existence of various polytypic forms as mentioned in our paper on the crystal structure of the 2-layer variant (Nishi \& Takéuchi, 1984a). It
appears that polytypism likewise exists in tricalcium silicate $\left(\mathrm{C}_{3} \mathrm{~S}\right)$; in addition to the rhombohedral structure, which may be interpreted as a 9 -layer structure (Takéuchi, Nishi \& Maki, 1984; Nishi \& Takéuchi, 1984b), the existence of a two-layer structure has been reported by Pérez-Méndez, Howie \& Glasser (1984).

In view of the common occurrence of polytypic forms in these chemical phases, we propose to denote $N$-layer variants by the following expressions: tricalcium germanate- $N\left(\right.$ or $\mathrm{C}_{3} \mathrm{G} N$ ) or tricalcium silicate$N\left(\right.$ or $\left.\mathrm{C}_{3} \mathrm{~S} N\right)$ (it is suggested that Roman numerals be used for $N$ ).
Although in general $\mathrm{C}_{3} \mathrm{G} N$ is isotypic with $\mathrm{C}_{3} \mathrm{~S} N$, the space group reported for $\mathrm{C}_{3} \mathrm{~S}$ II (Pérez-Méndez et al., 1984) is different from the one we found for $\mathrm{C}_{3}$ G II (Nishi \& Takéuchi, 1984a), the former being $P 6_{3} / m m c$ and the latter $P 6_{3} m c$. Note that in the structures of these chemical phases the germanate (or silicate) tetrahedron shows a trend towards orientational disorder; a fraction of the tetrahedron points up the $c$ axis (a $U$ orientation) while the remaining fraction points down the $c$ axis (a $D$ orientation). Slight differences in the occupancy between the pairs of differently oriented tetrahedra may cause such a difference in the space group although the structures are based on the same principle. Since, however, the structure determination of $\mathrm{C}_{3} \mathrm{~S}$ II (Pérez-Méndez et al., 1984) is based on a small number of diffraction intensities, further refinement of the structure is desirable; it is anticipated that the structure would possibly be strictly isotypic with our $\mathrm{C}_{3}$ G II. In any case, the theory of polytypism developed for $\mathrm{C}_{3} \mathrm{G}$ will be applicable to $\mathrm{C}_{3} \mathrm{~S}$.

The present paper describes the result of our structure analysis of a 24-layer variant of $\mathrm{C}_{3} \mathrm{G}\left(\mathrm{C}_{3} \mathrm{G}\right.$ XXIV) and discusses the general derivation of polytypes in $\mathrm{C}_{3} \mathrm{G}$.

## Experimental

Single crystals of tricalcium germanate were synthesized from a stoichiometric mixture of $\mathrm{Ca}_{2} \mathrm{GeO}_{4}$ and $\mathrm{CaF}_{2}$ plus small amounts of $\mathrm{Al}_{2} \mathrm{O}_{3}$ (about $2 \mathrm{wt} \%$ ). The mixture was melted at 1720 K and cooled to room temperature. X-ray examination of the product revealed the existence of $\mathrm{C}_{3} \mathrm{G}$ XXIV and $\mathrm{C}_{3} \mathrm{G}$ II in addition to $\mathrm{C}_{3} \mathrm{G}$ IX which was the major product. Electron microprobe analyses of the $\mathrm{C}_{3} \mathrm{G}$ XXIV crystals yielded: $\mathrm{CaO} 62 \cdot 19, \mathrm{GeO}_{2} 37 \cdot 07$ and $\mathrm{Al}_{2} \mathrm{O}_{3}$ $0.78 \mathrm{wt} \%$; the total was $100.04 \mathrm{wt} \%$. A crystal of $\mathrm{C}_{3} \mathrm{G}$ XXIV was ground to the shape of a sphere with diameter 0.18 mm . We determined the cell dimensions (see Abstract) from the $2 \theta$ values of 10 reflections ( $10 \cdot 2^{\circ} \leq 2 \theta \leq 34 \cdot 0^{\circ}$ ) measured with a Syntex $P 2_{1}$ four-circle single-crystal diffractometer using graphite-monochromated Mo $K \alpha$ radiation ( $\lambda=$ $0.71069 \AA$ ). In a precession photograph (Fig. 1) very weak extra reflections were observered, as in the case of $\mathrm{C}_{3}$ G II (Nishi \& Takéuchi, 1984a), suggesting that the crystal is not ideally of single phase but contaminated with very minor amounts of some other polytypic forms.

The diffraction intensities were measured with the $\omega-2 \theta$ scan technique up to $2 \theta=70^{\circ}$. One-twelfth Mo $K \alpha$ diffraction intensities $I(h k l)$ and another onetwelfth $I(\bar{h} \bar{k} \bar{l})$ were collected, the former giving a set of intensities of 524 reflections, while the latter that of 600 reflections. By averaging symmetrically equivalent reflection intensities, we obtained a set of 440 diffraction intensities, reflections whose intensities were smaller than $3 \sigma(I)$ being omitted.


Fig. 1. Precession photograph showing $h 0 l$ reflections of $\mathrm{C}_{3} \mathrm{G}$ XXIV (Mo $K \alpha$ ). Some of the extra reflections are indicated; $a_{h}^{*}$ and $c_{h}^{*}$ indicate the reciprocal axes of the hexagonal cell of the rhombohedral substructure.

The ranges of the indices were $0 \leq h \leq 10,0 \leq k \leq 9$ and $0 \leq l \leq 96$. The intensity fluctuations of three reference reflections were within $4 \cdot 4 \%$ (of $I$ value). The intensities were corrected for Lorentz, polarization and absorption effects (transmission factors: $\max .0 \cdot 38, \min .0 \cdot 32$ ) and reduced to structure factors.

## Structure determination

## Conditions of layer stacking

The unit layer for the polytypic series now considered is illustrated in Fig. 2. The unit cell of the layer, which has plane symmetry $P 3 m 1$, contains one germanate tetrahedron, three Ca atoms and one separate O atom, not associated with Ge , in its asymmetric unit. The possible stacking operations may be classified into two categories: (1) displacement vectors, $\mathbf{S}_{0}, \mathbf{S}_{1}$ and $\mathbf{S}_{2}$, with their horizontal components, $\mathbf{s}_{0}, \mathbf{s}_{1}$ and $\mathbf{s}_{2}$, as illustrated in Fig. 2, and (2) combined operations in which any one of $\mathbf{S}_{0}, \mathbf{S}_{1}$ and $\mathbf{S}_{2}$ is followed by a rotation $\varphi=(2 t+1) \times 60^{\circ}$ about an axis which may be represented by a line passing through the unit-cell origin and perpendicular to the layer, where $t$ is an integer. The operation in the latter category may hence be expressed by $\mathbf{S}_{0} \varphi$. Because of the high symmetry of the unit layer, however, we may consider only $\mathbf{S}_{0}$ in category (1) and $\mathbf{S}_{0} \varphi_{t=1}$ in (2). The displacements ( $\pm \mathbf{S}_{0}$ ), moreover, between any two successive layers can be decided in obedience to the orientation [ $0^{\circ}$ or $180^{\circ}\left(=\varphi_{t=1}\right)$ ] of the layer having lower position. Accordingly, one kind of character representing the orientational sequences is sufficient for the description. For simplicity, these orientational vectors will be represented by $\mathbf{1}\left[0^{\circ}\right]$ and $\overline{\mathbf{1}}\left[180^{\circ}(=\right.$ $\left.\left.\varphi_{t=1}\right)\right]$. This situation implies that possible polytypes


Fig. 2. Structure of the unit layer, showing that the unit cell contains three Ca atoms (small open circles), one separate O atom (large open circles) and one $\mathrm{GeO}_{4}$ tetrahedron. All tetrahedra are shown in the $U$ orientation. $\mathrm{s}_{0}, \mathrm{~s}_{1}$ and $\mathrm{s}_{2}$ represent horizontal components of the displacement vectors. In general, the tetrahedra may have $U-D$ orientational disorder.
of $\mathrm{C}_{3} \mathrm{G}$ (or $\mathrm{C}_{3} \mathrm{~S}$ ) may well be studied in terms of the sequence of $\mathbf{1}$ and $\overline{\mathbf{1}}$. The symbols $\mathbf{1}$ and $\overline{\mathbf{1}}$ will further be represented by + and - , respectively (see Guinier et al., 1984). Then the stacking sequence

$$
+++++\ldots=|+|_{3}
$$

gives a 3-layer rhombohedral polytype. And

$$
+-+-+\ldots=\mid+-1
$$

gives the 2-layer hexagonal structure.
Between the two symbols there is no restriction in the succession of + and - . Therefore, a layer + (or - ) may be followed by either + or - .

In the polytypes thus derived, the threefold axes and mirror planes of the unit layer are preserved. Accordingly, the structure of any polytype will be described by a hexagonal cell and have space group $P 3 m 1$ or one which has an order higher than that of $P 3 m 1$. Now consider the case in which $m$ layers define a periodicity perpendicular to the layers:

$$
\begin{equation*}
\overbrace{|+\ldots+-+\ldots--++\ldots+-\ldots+|}^{m} \tag{1}
\end{equation*}
$$

Suppose that layers, - , occur in this sequence at the $Q_{1}$ th, $Q_{2}$ th, $\ldots, Q_{n}$ th layers, where $Q_{1}<Q_{2}<Q_{3}<$ $\ldots<Q_{n}(n \leq m)$. Then the horizontal shift $X$ of all the atoms in the $(m+1)$ th layer relative to that of the first layer is given by

$$
\begin{equation*}
\mathbf{X}=(m-2 n) * \mathbf{s}_{0} . \tag{2}
\end{equation*}
$$

Then, once the layer number $m$ is given, the possible stacking sequence can be derived from the result of (2).

In the case of a 24-layer structure having rhombohedral symmetry, we may consider the mode of stacking of only eight layers, namely $m=8(=24 / 3)$. We then solved equation (2) under the condition $[m-2 n \equiv 1(\bmod 3), 0 \leq n \leq 8, n:$ integer] and derived many sets of sequences from its solution, the result giving ten independent sets of sequences. Preliminary calculations of structure factors of these ten structure models revealed that one gave a relatively low $R$ value of $46 \%$ compared with those of the others, $54 \% \sim$ $70 \%$. This model, having the stacking sequence
 be shown below.

## Refinement of the structure

Since the $U-D$ orientational disorder of tetrahedra commonly occurs in $\mathrm{C}_{3}$ S IX (Takéuchi et al., 1984; Nishi \& Takéuchi, 1984b) and C ${ }_{3}$ G II (Nishi \& Takéuchi, 1984a), the refinement of the above model was initiated by carrying out a Fourier synthesis based on structure factors calculated from the positional parameters of $\mathrm{Ca}, \mathrm{Ge}$ and separate O atoms, the O atoms about Ge being omitted. In the resulting Fourier map, the mode of $U-D$ orientational disorder

Table 1. Atomic parameters for 24-layer tricalcium germanate (values of the positional parameters are multiplied by $10^{4}$ )

|  | Occupancy | $x$ | $y$ | $z$ | $B\left(\AA^{2}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Ca}(1)$ | 1.0 | 5105 (20) | -5105 (20) | 36 (6) | $2 \cdot 8(5)$ |
| $\mathrm{Ca}(2)$ | 1.0 | 8112 (17) | -8112 (17) | 467 (5) | $2 \cdot 3$ (4) |
| $\mathrm{Ca}(3)$ | 1.0 | 5142 (20) | -5142 (20) | 856 (6) | $4 \cdot 0$ (6) |
| $\mathrm{Ca}(4)$ | 1.0 | 1769 (18) | -1769 (18) | 1243 (5) | $2 \cdot 4$ (4) |
| $\mathrm{Ca}(5)$ | 1.0 | 4850 (15) | -4850 (15) | 1678 (5) | 2.0 (4) |
| $\mathrm{Ca}(6)$ | 1.0 | 1716 (15) | -1716 (15) | 2080 (5) | 1.8 (3) |
| $\mathrm{Ca}(7)$ | 1.0 | 8411 (21) | -8411 (21) | 2520 (6) | $3 \cdot 8(5)$ |
| $\mathrm{Ca}(8)$ | 1.0 | 1532 (25) | -1532 (25) | 2938 (6) | $3 \cdot 8$ (6) |
| $\mathrm{Ge}(11)$ | $0 \cdot 56$ (4) | 0 | 0 | 0 | 1.7 (5) |
| $\mathrm{Ge}(21)$ | 0.79 (4) | 0 | 0 | 7087 (5) | $1 \cdot 2(4)$ |
| $\mathrm{Ge}(31)$ | 0.56 (4) | 0 | 0 | 861 (9) | 1.0 (3) |
| $\mathrm{Ge}(41)$ | 0.79 (4) | 0 | 0 | 4638 (5) | $1 \cdot 3$ (4) |
| $\mathrm{Ge}(51)$ | 0.56 (4) | 0 | 0 | 1643 (5) | 0.4 (3) |
| $\mathrm{Ge}(61)$ | 0.79 (4) | 0 | 0 | 5476 (6) | 1.7 (4) |
| $\mathrm{Ge}(71)$ | 0.56 (4) | 0 | 0 | 9265 (7) | $2 \cdot 1$ (6) |
| $\mathrm{Ge}(81)$ | 0.79 (4) | 0 | 0 | 6273 (6) | $2 \cdot 0$ (6) |
| $\mathrm{Ge}(12)$ | 0.44 (4) | 0 | 0 | 102 (8) | 1.7 (5) |
| $\mathrm{Ge}(22)$ | 0.21 (4) | 0 | 0 | 7121 (15) | 1.2(4) |
| $\mathrm{Ge}(32)$ | 0.44 (4) | 0 | 0 | 852 (8) | 1.0 (3) |
| $\mathrm{Ge}(42)$ | 0.21 (4) | 0 | 0 | 4588 (14) | 1.3 (4) |
| $\mathrm{Ge}(52)$ | 0.44 (4) | 0 | 0 | 1703 (5) | 0.4 (3) |
| $\mathrm{Ge}(62)$ | 0.21 (4) | 0 | 0 | 5465 (15) | 1.7 (4) |
| $\mathrm{Ge}(72)$ | 0.44 (4) | 0 | 0 | 9200 (8) | $2 \cdot 1$ (6) |
| $\mathrm{Ge}(82)$ | 0.21 (4) | 0 | 0 | 6244 (18) | $2 \cdot 0$ (6) |
| $\mathrm{O}(1)$ | 1.0 | 0 | 0 | 3605 (33) | 8 (5) |
| O(2) | 1.0 | 0 | 0 | 4028 (33) | 4 (4) |
| $\mathrm{O}(3)$ | 1.0 | 0 | 0 | 7591 (32) | 6 (6) |
| $\mathrm{O}(4)$ | 1.0 | 0 | 0 | 8120 (27) | 3 (3) |
| $\mathrm{O}(5)$ | 1.0 | 0 | 0 | 8563 (32) | 6 (4) |
| $\mathrm{O}(6)$ | 1.0 | 0 | 0 | 2333 (30) | 6 (3) |
| O(7) | 1.0 | 0 | 0 | 2744 (27) | 4 (4) |
| $\mathrm{O}(8)$ | 1.0 | 0 | 0 | 3188 (13) | 1 (2) |
| $\mathrm{OA}(11)$ | 0.56 (4) | 0 | 0 | 262 | 2 (2) |
| OA(21) | 0.79 (4) | 0 | 0 | 7349 | 11 (6) |
| OA(31) | 0.56 (4) | 0 | 0 | 1123 | 7 (3) |
| OA(41) | 0.79 (4) | 0 | 0 | 4900 | -1(1) |
| OA(51) | 0.56 (4) | 0 | 0 | 1905 | S(3) |
| OA(61) | 0.79 (4) | 0 | 0 | 5738 | 1(1) |
| OA(71) | 0.56 (4) | 0 | 0 | 9527 | 2(2) |
| OA(81) | 0.79 (4) | 0 | 0 | 6535 | $5(2)$ |
| OA(12) | 0.44 (4) | 0 | 0 | -160 | 2(2) |
| OA(22) | 0.21 (4) | 0 | 0 | 6859 | 11(6) |
| OA(32) | 0.44 (4) | 0 | 0 | 590 | $7(3)$ |
| $\mathrm{OA}(42)$ | 0.21 (4) | 0 | 0 | 4326 | -1(1) |
| OA(52) | 0.44 (4) | 0 | 0 | 1441 | 5(3) |
| OA(62) | 0.21 (4) | 0 | 0 | 5203 | 1(1) |
| OA(72) | 0.44 (4) | 0 | 0 | 8938 | 2(2) |
| $\mathrm{OA}(82)$ | 0.21 (4) | 0 | 0 | 5982 | $5(2)$ |
| OB(11) | 0.56 (4) | 8667 | -8667 | -88 | 2(2) |
| $O B(21)$ | 0.79 (4) | 1333 | -1333 | 6999 | 11(6) |
| OB(31) | 0.56 (4) | 8667 | -8667 | 773 | 7(3) |
| OB(41) | 0.79 (4) | 8667 | -8667 | 4551 | -1(1) |
| $O B(51)$ | 0.56 (4) | 1333 | -1333 | 1556 | $5(3)$ |
| $O B(61)$ | 0.79 (4) | 8667 | -8667 | 5389 | 1(1) |
| $O B(71)$ | 0.56 (4) | 8667 | -8667 | 9177 | 2(2) |
| $\bigcirc B(81)$ | 0.79 (4) | 1333 | -1333 | 6185 | $5(2)$ |
| $\bigcirc B(12)$ | 0.44 (4) | 8667 | -8667 | 189 | 2(2) |
| $O B(22)$ | 0.21 (4) | 1333 | -1333 | 7209 | $11(6)$ |
| $O B(32)$ | 0.44 (4) | 8667 | -8667 | 940 | 7(3) |
| $O B(42)$ | 0.21 (4) | 8667 | -8667 | 4676 | -1(1) |
| $O B(52)$ | 0.44 (4) | 1333 | -1333 | 1790 | $5(3)$ |
| $O B(62)$ | 0.21 (4) | 8667 | -8667 | 5553 | 1(1) |
| OB(72) | 0.44 (4) | 8667 | -8667 | 9288 | 2(2) |
| OB(82) | 0.21 (4) | 1333 | -1333 | 6332 | 5 (2) |

of each tetrahedron was unambiguously revealed. For further isotropic refinement, including the occupancy parameters of a $U-D$ pair of each tetrahedron, the configuration of the tetrahedra was constrained to the shape of a regular tetrahedron with Ge-O $1.77 \AA$ and $\angle \mathrm{O}-\mathrm{Ge}-\mathrm{O} 109 \cdot 5 .^{\circ}$ For the calculations, we used the least-squares program LINUS (Coppens \& Hamilton, 1970) and neutral-atom scattering factors with correction for anomalous dispersion taken from International Tables for $X$-ray Crystallography (1974). Unit weights were used.

The final residual factors were $R=0 \cdot 113$ and $w R=$ $0 \cdot 104$ and the ratios of average and maximum shifts to error were 0.3 and 1.5 , respectively. A difference map showed that residual electron density ranged from -1.9 to $2 \cdot 1 \mathrm{e} \AA^{-3}$. Table 1 gives the final atomic coordinates and thermal parameters.*

## Discussion

## Detailed structure

The projection of the $\mathrm{C}_{3} \mathrm{G}$ XXIV structure along [110] thus obtained is illustrated schematically in Fig. 3 together with the corresponding projections of the structures of $\mathrm{C}_{3}$ G II (Nishi \& Takéuchi, 1984a) and $\mathrm{C}_{3}$ G IX (Nishi \& Takéuchi, 1985). A comparison of these three structures shows that the 24-layer polytype consists of two kinds of portions; one is related to the structure of the 2-layer polytype and the other to that of the 9 -layer polytype which is most commonly observed in $\mathrm{C}_{3} \mathrm{G}$ and $\mathrm{C}_{3} \mathrm{~S}$. This situation can be readily expressed in terms of stacking symbols as shown below:


[^0]

Fig. 3. Schematic drawings of three different polytypes. The arrangement shows the Ca atoms (small circles), separate O atoms (large circles) and $\mathrm{GeO}_{4}$ tetrahedra (solid triangles) in the mirror plane passing through the long diagonal of the hexagonal cell of each structure. All tetrahedra are drawn in the $U$ orientation. In the $\mathrm{C}_{3} \mathrm{G}$ XXIV structure, the Roman letters 'II' and 'IX' represent a full structure of $\mathrm{C}_{3} \mathrm{G}$ II and a partial structure of $\mathrm{C}_{3} \mathrm{G}$ IX respectively. The solid lines trace the unit cells [the origin of the 2-layer structure is shifted by $\frac{1}{3}\left(\mathbf{a}_{1}-\mathbf{a}_{2}\right)$ from the true origin].
['II' means the structure of the 2-layer polytype and 'IX'*(-1) means that the structure of the 9 -layer one is rotated by $180^{\circ}$ about the $c$ axis.]

The coexistence of these polytypes in the same product of a run would be related to such a structural relationship among these polytypes. Detailed features of the $\mathrm{C}_{3} \mathrm{G}$ XXIV structure are shown in Fig. 4.

The $\mathrm{Ca}-\mathrm{O}$ bond lengths of each polyhedon are given in Table 2. For the evaluation of the mean $\mathrm{Ca}-\mathrm{O}$ bond length for each polyhedron, each $\mathrm{Ca}-\mathrm{O}$ bond length was weighted by its bond occupancy. The $\mathrm{Ca}-\mathrm{O}$ mean values are in the range 2.34 to $2.51 \AA$, giving an overall mean value of $2 \cdot 42 \AA$. The coordination number of each Ca atom was counted by summing the occupancy of each $\mathrm{Ca}-\mathrm{O}$ bond length; the coordination numbers range from 6.0 to 6.56 , with a mean value of $6 \cdot 15$. The above overall mean of the $\mathrm{Ca}-\mathrm{O}$ bond lengths is larger than the corresponding value of $2.38 \AA$ found for $\mathrm{C}_{3}$ G II (Nishi \& Takéuchi, 1984a). This is because F atoms, having a smaller ionic radius than O , partially substitute for the separate O atoms in $\mathrm{C}_{3} \mathrm{G}$ II (Nishi \& Takéuchi, 1984a).

It is notable that each position for Ge is more or less split into a pair of positions. In order to express


Fig. 4. The crystal structure of $\mathrm{C}_{3} \mathrm{G}$ XXIV showing the germanate tetrahedra, separate O atoms and Ca atoms (and their anion neighbors) in the mirror plane passing through the long diagonal of the hexagonal cell. The $\mathrm{GeO}_{4}$ tetrahedra having occupancies of 56 and $79 \%$ are indicated by solid lines, those with occupancies of 44 and $21 \%$ by broken lines. The sizes of each atom are not proportional to real isotropic factors.

Table 2. $\mathrm{Ca}-\mathrm{O}$ bond lengths ( $\AA$ )

|  |  | $\mathrm{Ca}-\mathrm{O}$ bond |  |  |  |  |  | Ca-O bond |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { Symmetry } \\ & \text { code } \end{aligned}$ | Occupancy | Length |  |  | Symmetry code | Occupancy | Length |
| $\mathrm{Ca}(1)$ | $\bigcirc B^{(21)}$ | i | 0.79 | 2.07 (3) | $\mathrm{Ca}(5)$ | OB(42) | viii | 0.21 | 2.34 (3) |
|  | $\mathrm{O}^{\mathrm{O}} 8$ ( 8 | ii | 1.0 | 2.31 (5) |  | OB(52) | xiii | 0.44 | $2 \cdot 39$ (3) |
|  | $\bigcirc B(11)$ | iii | 0.56 | 2.43 (3) |  | OB(52) | xiv | 0.44 | 2.39 (3) |
|  | OB(11) | iv | 0.56 | 2.43 (3) |  | OA(41) | ii | 0.79 | $2 \cdot 40(3)$ |
|  | OA(22) | i | 0.21 | 2.46 (3) |  | O(5) | Ir | 1.0 | $2 \cdot 40(3)$ $2 \cdot 40(13)$ |
|  | OA(81) | i | 0.79 | $2 \cdot 49$ (3) |  | $\bigcirc B(51)$ | xiii | 0.56 | $2.40(13)$ $2 \cdot 41$ (3) |
|  | $\bigcirc{ }^{\circ}(12)$ | iii | 0.44 | 2.50 (3) |  | $\bigcirc \mathrm{B}(51)$ | xiv | 0.56 | 2.41 (3) 2.41 (3) |
|  | $\mathrm{OB}(12)$ | iv | 0.44 | 2.50 (3) |  | $\mathrm{O}(4)$ | xiv | 1.0 | $2.41(3)$ $2.43(12)$ $2.62(3)$ |
|  | $\mathrm{O}(1)$ | ii | 1.0 | 2.52 (14) |  | OB(61) | viii | 0.79 | 2.43 (12) 2.62 (3) |
|  | $\mathrm{OB}(82)$ | $i$ | 0.21 | 2.56 (3) |  | OB(61) OA(62) | ${ }_{\text {iiii }}^{\text {viii }}$ | 0.79 0.21 | $2.62(3)$ $2.62(3)$ |
|  | Total |  | 6.0 |  |  | Total |  | 6.0 | $2 \cdot 62$ (3) |
|  | Average |  |  | 2.40 |  | Average |  | 6.0 | 2.44 |
| $\mathrm{Ca}(2)$ | $O B(12)$ |  | 0.44 | 2.00 (3) | Ca (6) | OB(52) |  |  |  |
|  | $O B(31)$ |  | 0.56 | 2.18(3) | Ca(6) | $\bigcirc{ }^{\text {O }}$ (61) | ix | 0.44 0.79 | $2.01(3)$ $2.31(3)$ |
|  | O (1) | ii | 1.0 | $2 \cdot 24$ (13) |  | OB(61) | ix | 0.79 0.79 | $2.31(3)$ $2.31(3)$ |
|  | $\mathrm{OB}(22)$ | $v$ | $0 \cdot 21$ | $2 \cdot 30$ (3) |  | $\mathrm{O}(5)$ | $x$ | 0.79 1.0 | 2.31(3) $2 \cdot 38(11)$ |
|  | $\bigcirc \mathrm{O}(22)$ | vi | 0.21 | $2 \cdot 30$ (3) |  | OA(72) | i | 1.0 0.44 | $2.38(11)$ 2.40 (3) |
|  | $\mathrm{O}(2)$ | ii | 1.0 | 2.37 (15) |  | OA(51) | 1 | 0.44 0.56 | $2.40(3)$ $2.45(3)$ |
|  | OB(21) | $v$ | 0.79 | 2.42 (3) |  | $\bigcirc B(62)$ | ix | 0.56 0.21 | $2.45(3)$ 2.49 (3) |
|  | OB(21) | vi | 0.79 | 2.42 (3) |  | $\bigcirc B(62)$ | ${ }^{\text {ix }}$ | 0.21 0.21 | 2.49 (3) 2.49 (3) |
|  | OA(32) | vii | 0.44 | 2.51 (3) |  | O(6) | x | 0.21 1.0 | 2.49 (3) $2.74(12)$ |
|  | OA(11) | vii | 0.56 | 2.74 (3) |  | OB(71) | xv | 1.0 0.56 | $2.74(12)$ $2.92(3)$ |
|  | Total |  | 6.0 |  |  | Total |  | 6.0 | 2.92 (3) |
|  | Average |  |  | 2.36 |  | Average |  |  | 2.46 |
| $\mathrm{Ca}(3)$ | $\mathrm{OA}(42)$ | ii | 0.21 1.0 | $2.12(3)$ $2.20(1)$ | $\mathrm{Ca}(7)$ | OB(71) | xvi | 0.56 | 2.29(3) |
|  | $\mathrm{O}(2)$ | ii | 1.0 | 2.20(11) |  | OB(71) | xvii | 0.56 | 2.29 (3) |
|  | OB(22) | i | 0.21 | 2.20 (3) |  | $\bigcirc O^{(81)}$ | ii | 0.79 | 2.30 (3) |
|  | $\mathrm{O}(3)$ | iii | 1.0 | 2.31 (5) |  | OA(61) | ii | 0.79 | 2.32(3) |
|  | OB(31) | iii | 0.56 | 2.33(3) |  | OA(82) | ii | 0.21 | $2 \cdot 35$ (3) |
|  | $\bigcirc B(31)$ | iv | 0.56 | 2.33 (3) |  | $\mathrm{O}(6)$ | vii | 1.0 | $2 \cdot 36$ (11) |
|  | $\bigcirc B(32)$ | iii | 0.44 | 2.34 (3) |  | OB(72) | xvi | 0.44 | $2 \cdot 39$ (3) |
|  | OB(32) | iv | 0.44 | 2.34 (3) |  | OB(72) | xvii | $0 \cdot 44$ | $2 \cdot 39$ (3) |
|  | OB(41) | viii | 0.79 | $2 \cdot 45$ (3) |  | $\mathrm{O}(7)$ | vii | 1.0 | 2.50(11) |
|  | OA(21) | i | 0.79 | 2.55 (3) |  | OB(62) | xviii | $0 \cdot 21$ | 2.83 (3) |
|  | Total Average |  | 6.0 |  |  | $\bigcirc B(62)$ | xix | 0.21 | 2.83(3) |
|  |  |  |  | 2.34 |  | Total |  | $6 \cdot 21$ |  |
| $\mathrm{Ca}(4)$ |  |  | 0.56 |  |  | Average |  |  | 2.39 |
|  | $\bigcirc B(41)$ | ix | 0.79 | $2.18(3)$ $2.29(3)$ |  |  |  |  |  |
|  | OB(41) | x | 0.79 | 2.29 (3) | $\mathrm{Ca}(8)$ | $O B(11)$ $O B(72)$ | xx xv c | 0.56 0.44 | $2 \cdot 16(3)$ $2 \cdot 21(3)$ |
|  | OA(31) |  | 0.56 | 2.36(3) |  | $\bigcirc{ }^{\circ}(82)$ | xxi | 0.21 | 2.31 (3) |
|  | OB(42) | ${ }^{\text {ix }}$ | 0.21 | 2.38 (3) |  | OB(82) | rxii | 0.21 | 2.31 (3) |
|  | $\bigcirc B(42)$ | x | 0.21 | 2.38 (3) |  | OA(71) | , | 0.56 | 2.31 (3) |
|  | $\mathrm{O}(4)$ | i | ${ }^{1.0}$ | 2.42 (11) |  | $\mathrm{O}(7)$ |  | 1.0 | 2.32(11) |
|  | OA (52) |  | 0.44 | 2.58 (3) |  | OB(81) | , | 0.79 | $2 \cdot 34$ (3) |
|  | $\bigcirc B(32)$ | ${ }_{\text {xi }}$ | 0.44 | 2.86 (3) |  | OB(81) | xxii | 0.79 | 2.34 (3) |
|  | OB(32) | xii | 0.44 | 2.86 (3) |  | $\mathrm{O}(8)$ |  | 1.0 | 2.56 (7) |
|  | $\mathrm{O}(3)$ | i | 1.0 6.44 | 2.91 (16) |  | $\bigcirc A^{(12)}$ | xxiii | 0.44 | 2.76 (3) |
|  | Total <br> Average |  | 6.44 |  |  | $\bigcirc B(71)$ | $\mathrm{x} v$ | 0.56 | 2.94 (3) |
|  |  |  |  | $2 \cdot 51$ |  | Total |  | 6.56 |  |
|  |  |  |  |  |  | Average |  |  | 2.42 |

Symmetry code

| None | $x, y, z$ |
| :--- | :--- |
| (iii) | $-y, x-y-2, z$ |
| (vi) | $-x+y+\frac{4}{3},-x-\frac{1}{3}, z-\frac{2}{3}$ |
| (ix) | $-y-\frac{1}{3}, x-y-\frac{5}{3}, z-\frac{1}{3}$ |
| (xii) | $-x+y+2,-x+1, z$ |
| (xv) | $x-\frac{2}{3}, y+\frac{2}{3}, z-\frac{2}{3}$ |
| (xnii) | $-y-1 \frac{1}{3}, x-y-\frac{8}{3}, z-\frac{1}{3}$ |
| (xxi) | $-y-\frac{1}{3}, x-y-\frac{2}{3}, z-\frac{1}{3}$ |


| (i) | $x+\frac{1}{3}, y-\frac{1}{3}, z-\frac{2}{3}$ |
| :--- | :--- |
| (iv) | $-x+y+2,-x, z$ |
| (vii) | $x+1, y-1, z$ |
| (x) | $-x+y+\frac{5}{3},-x+\frac{1}{3}, z-\frac{1}{3}$ |
| (xiii) | $-y, x-y-1, z$, |
| (xvi) | $-y+\frac{1}{3}, x-y-\frac{7}{3}, z-\frac{2}{3}$ |
| (xix) | $-x+y+\frac{8}{3},-x+\frac{1}{3}, z-\frac{1}{3}$ |
| (xxii) | $-x+y+\frac{1}{3},-x+\frac{1}{3}, z-\frac{1}{3}$ |

Table 3. Splitting distances ( $\AA$ ) of Ge atoms

| $\mathrm{Ge}(11)-\mathrm{Ge}(12)$ | $0.69(5)$ | $\mathrm{Ge}(51)-\mathrm{Ge}(52)$ | $0.40(5)$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{Ge}(21)-\mathrm{Ge}(22)$ | $0.23(11)$ | $\mathrm{Ge}(61)-\mathrm{Ge}(62)$ | $0.07(11)$ |
| $\mathrm{Ge}(31)-\mathrm{Ge}(32)$ | $0.34(10)$ | $\mathrm{Ge}(71)-\mathrm{Ge}(72)$ | $0.44(7)$ |
| $\mathrm{Ge}(41)-\mathrm{Ge}(42)$ | $0.06(8)$ | $\mathrm{Ge}(81)-\mathrm{Ge}(82)$ | $0.20(13)$ |

such a situation, the pair of positions of the $j$ th Ge , namely $\mathrm{Ge}(j)$, is expressed by $\mathrm{Ge}(j 1)$ and $\mathrm{Ge}(j 2)$ (Table 3). In particular, the separations of $\mathrm{Ge}(1)$, $\mathrm{Ge}(3), \mathrm{Ge}(5), \mathrm{Ge}(7)$ are conspicuous, the values being in the range $0.34(10)$ to $0.69(5) \AA$; in each case the
occupancies of the pair positions are nearly the same. For the remaining Ge atoms, the separations are in the range $0.06(8)$ to $0.23(11) \AA$ and the occupancies of the pair positions of these Ge atoms are significantly different (Table 1).
The above features of the positional splittings of the Ge atoms are well in line with the mode of $U-D$ orientational disorder of the Ge tetrahedra. The tetrahedra about the four above-mentioned Ge atoms show nearly perfect $U-D$ disorder, while the remaining ones only partial.

Table 4. Possible simple polytypes of $C_{3} G$ and $C_{3} S$ (only one example is given in each case of the 9-and 24-layer polytypes)

In each compound the known polytypes are indicated by asterisks.
Polytype
Compounds having related structures

| Symbol | Layer sequence | Space group | $\mathrm{G}_{3} \mathrm{G}$ | $\mathrm{C}_{3} \mathrm{~S}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 H | $1+-1$ | $\mathrm{Pb}_{3} \mathrm{mc}$ | *[1] | *[2] | $\left(\mathrm{Cd}_{0.5} \mathrm{~Pb}_{0.5}\right)_{3} \mathrm{SiO}_{5}[3]$ |
| $3 R$ | $1+13$ | R3m |  |  | $\mathrm{Cd}_{3} \mathrm{SiO}_{5}[4], \mathrm{Pb}_{3} \mathrm{SiO}_{5}$ [5], $\mathrm{Ba}_{3} \mathrm{SiO}_{5}[6], \mathrm{Sr}_{3} \mathrm{SiO}_{5}[7], \mathrm{Pb}_{3} \mathrm{GeO}_{5}[8]$ |
| 4H | $1+--+1$ | $P^{P 6}{ }_{3} m c$ |  |  |  |
| $5 T$ | $1+-+++1$ | P3m1 |  |  |  |
| 6 H | $1+---++1$ | $\mathrm{Pr}_{3} \mathrm{mc}$ |  |  |  |
| $6 T$ | $1+--+-+1$ | ${ }_{\text {P }} \times 2 \mathrm{ml}$ |  |  |  |
| $9 R$ | $1+-+1_{3}$ | R3m | *[9] | *10] | solution of $\mathrm{C}_{3} \mathrm{G}$ and $\mathrm{C}_{3} \mathrm{~S}$ [14] |
| $24 R$ | $1+--+--+-1_{3}$ | R3m | *[15] |  |  |

References: [1] Nishi \& Takéuchi (1984a), [2] Pérez-Méndez et al. (1984), [3] Eysel \& Breuer (1983), [4] Eysel (1970), [5] Ott \& MacLaren (1970), [6] Tillmanns \& Grosse (1978), [7] Dent Glasser \& Glasser (1965), [8] Otto (1979), [9] Nishi \& Takéuchi (1985), [10] Nishi \& Takéuchi (1984b), [11] Takéuchi et al. (1984), [12] Golovastikov et al. (1975), [13] Bigaré et al. (1967), [14] Eysel \& Hahn (1970), [15] this work.

## Other polytypes

The polytypes of $\mathrm{C}_{3} \mathrm{G}, \mathrm{C}_{3} \mathrm{~S}$ and $M_{3} \mathrm{~S}(M=\mathrm{Cd}, \mathrm{Pb}$, $\mathrm{Ba}, \mathrm{Sr}$ ) which are known to date are 2-layer, 3-layer, 9 -layer and 24-layer structures. In addition to these known polytypes, we may now theoretically derive, using relation (2), the possible series of polytypes of these chemical phases. Some simple cases are summarized in Table 4. Among them, the structures of 4-, 5-, and 6-layer polytypes are illustrated in Fig. 5.

Finally, it is notable that a layer similar to our unit layer has been found in a new mineral, arctite (Egorov-Tismenko, Sokolova \& Smirnova, 1984). Although the layer consists of $\mathrm{Ca}, \mathrm{P}$ and O , it shares


Fig. 5. Predicted structures of some simple polytypes. (a) 4H, (b) $5 T$, (c) $6 H$ [the origin is shifted by $\frac{1}{3}\left(a_{2}-a_{1}\right)$ from the true origin] and (d) $6 T$. The structures are represented as those in Fig. 3.
geometrical features with the present unit layer. It hence appears that such a layer configuration is common among silicates, germanates and phosphates having large cations sich as $\mathrm{Ca}^{2+}$.

## References

Bigaré, M., Guinier, A., Mazières, C., Regourd, M., Yannaquis, N., Eysel, W., Hahn, Th. \& Woermann, E. (1967). J. Am. Ceram. Soc. 50, 609-619.

Coppens, P. \& Hamilton, W. C. (1970). Acta Cryst. A26, 71-83.
Dent Glasser, L. S. \& Glasser, F. P. (1965). Acta Cryst. 18, 453-457.
Egorov-Tismenko, Yu. K., Sokolova, E. V. \& Smirnova, N. L. (1984). Acta Cryst. A40, C245.

Eysel, W. (1970). Neues Jahrb. Mineral. Monatsh, pp. 534-547.
Eysel, W. \& Breuer, K. H. (1983). Z. Kristallogr. 163, 1-17.
Eysel, W. \& Hahn, Th. (1970). Z. Kristallogr. 131, 40-59.
Golovastikov, R., Matveeva, R. G. \& Belov, N. V. (1975). Sov. Phys. Crystallogr. 20, 441-445.
Guinier, A, Bokij, G. B., Boll-Dornberger, K., Cowley, J. M., Du brovič, S., Jagodzinski, H., Krishna, P., de Wolff, P. M., Zvyagin, B. B., Cox, D. E., Goodman, P., Hahn, Th., Kuchitsu, K. \& Abrahams, S. C. (1984). Acta Cryst. A40, 399-404.
International Tables for X-ray Crystallography (1974). Vol. IV. Birmingham: Kynoch Press. (Present distributor D. Reidel, Dordrecht.)
Nishi, F. \& Takéuchi, Y. (1984a). Acta Cryst. C40, 730-733.
Nishi, F. \& TAKÉUCHi, Y. (1984b). Z. Kristallogr. 168, 197-212.
Nishi, F. \& Takéuchi, Y. (1985). In preparation.
Ott, W. R. \& Maclaren, M. G. (1970). J. Am. Ceram. Soc. 53, 374-375.
Отто, Н. Н. (1979). Z. Kristallogr. 149, 227-240.
Pérez-Méndez, M., Howie, R. A. \& Glasser, F. P. (1984). Cem. Concr. Res. 14, 57-63.
Takéuchi, Y., Nishi, F. \& Maki, I. (1984). Acta Cryst. A40, C215.
Tillmanns, E. \& Grosse, H. P. (1978). Acta Cryst. B34, 649-651.


[^0]:    * A list of structure factors has been deposited with the British Library Lending Division as Supplementary Publication No. SUP $42245(4 \mathrm{pp}$.). Copies may be obtained through The Executive Secretary, International Union of Crystallography, 5 Abbey Square, Chester CH1 2HU, England.

